ALGEBRAIC TOPOLOGY I WS23/24, HOMEWORK SHEET 2

DEADLINE: FRIDAY, OCTOBER 27TH

Problem 1.

(1) Suppose that $S^n \to E \to B$ is a fibre sequence, $n \neq 0$, B simply connected. Show that there is a long exact sequence of the form

$$\ldots \to H_{p-n}(B) \to H_p(E) \to H_p(B) \to H_{p-n-1}(B) \to H_{p-1}(E) \to \ldots.$$

This sequence is called the *Gysin sequence* of the sphere bundle.

(2) Let $F \to E \to S^n$ be a fibre sequence over a sphere with $n \neq 0, 1$. Show that there exists a long exact sequence of the form

$$\ldots \to H_q(F) \to H_q(E) \to H_{q-n}(F) \to H_{q-1}(F) \to H_{q-1}(E) \to .$$

This sequence is called the *Wang sequence*.

Problem 2. Let X be a simply connected space which is not weakly homotopy equivalent to a point. Prove that it is not possible for both X and ΩX to have the homotopy type of a finite CW complex.

- Hint 1: Use the Hurewicz theorem to find a prime p such that both $\widetilde{H}_*(X, \mathbb{F}_p)$ and $\widetilde{H}_*(\Omega X, \mathbb{F}_p)$ are nontrivial.
- Hint 2: Consider the fibre sequence $\Omega X \to * \to X$.